TYPE 704: SLAB ON GRADE (INTERFACES WITH ZONE AIR TEMPERATURE)

General Description

This routine models the heat transfer from a horizontal surface (slab) to the soil beneath the surface. The heat transfer is assumed to be conductive only and moisture effects are not accounted for in the model. The model relies on a 3-dimensional finite difference model of the soil and solves the resulting inter-dependent differential equations using a simple iterative method. The user enters the temperature of the zone air above the slab, the thermal properties of the slab, the soil properties and grid geometry, and the ambient conditions outside of the slab (near field). The initial soil conditions may be calculated from the Kasuda correlation [1] or read from a user-supplied data file. The surface conditions for the near-field and far-field soil may be calculated in one of three ways; 1) from the Kasuda correlation, 2) from an energy balance on the surface plane, 3) provided as an input to the model. The near-field soil temperatures are affected by the heat transfer from the slab. The far-field soil temperatures are only affected by the surface conditions (time of year) and depth.

This radiant slab model is intended to be used with simple building models that calculate the zone air temperature and not with detailed building models that are capable of calculating inner surface temperatures based on provided outer surface temperatures (such as the standard TRNSYS Type 56 model). If you wish to use this radiant slab model in conjunction with Type56, please refer to the companion model, Type703.

Nomenclature

m	-	Mass.
C_p	-	Specific heat.
$\dot{Q}_{in}$	-	The rate at which energy enters a fluid, soil or slab node.
$\dot{Q}_{out}$	-	The rate at which energy exits a fluid, soil or slab node.
$T_{x,y,z}$	-	The temperature of a given soil node.
x	-	The distance along the length direction.
y	-	The distance along the width direction.
z	-	The depth dimension (node 1 is at the surface, increasing downwards).
k	-	Thermal conductivity.
dx	-	The x direction dimension of a soil or slab node.
dy	-	The y direction dimension of a soil or slab node.
dz	-	The z direction dimension of a soil or slab node.
$T_{surface}$	-	The surface temperature of a node exposed to ambient conditions.

Detailed Description

1. Appropriate Building Models

Simplified building models such as the lumped capacitance or degree-day models (for example Types 660, 80, and 12 in TRNSYS) do not contain the concept of a boundary wall and do not perform radiation exchange calculations to determine interior surface temperatures of all walls, floors and ceilings. Because of these simplifications one cannot use the same slab model with simplified building models and with the detailed Type56 building model; dedicated models are required for each type of building.

Type704 interfaces with the zone air temperature, performing an energy balance on the inner slab surface to determine energy input from the slab to the zone and, consequently, energy conducted to the soil below. The energy stored in the slab is not accounted for in this model as the capacitance of the slab is small when compared to the capacitance of the soil nodes beneath the slab.

2. Slab and Surroundings

Type704 divides the slab and its surroundings into three parts. First is the slab itself, for which the user specifies length and width as shown in Figure 1.

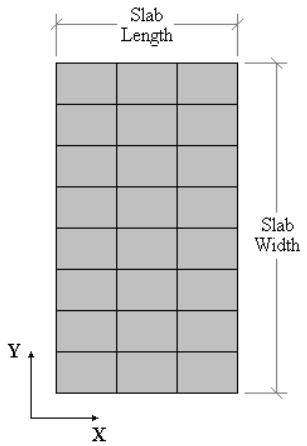


Figure 1: Slab Dimensions and Coordinate System

The slab is assumed to be sitting on top of soil, for which the user must specify the thermal properties. It is assumed that the edge of the slab is adiabatic, and that consequently, no heat transfer occurs between the edges of the slab and the surroundings. The soil volume surrounding the slab in the x, y and z directions as shown in Figure 2 is referred to as the near field. Nodes contained in the near-field can vary in size in all three dimensions (user-specified), usually becoming larger as they get farther from the slab, and are included in the slab energy balance. The near field is in turn surrounded by the far field, which is assumed to be an infinite energy sink/source (energy transfer with the far-field does not result in a temperature change of the far-field). Users have a number of options as to how they wish to treat the soil in both the near and far fields as discussed in Section 3.

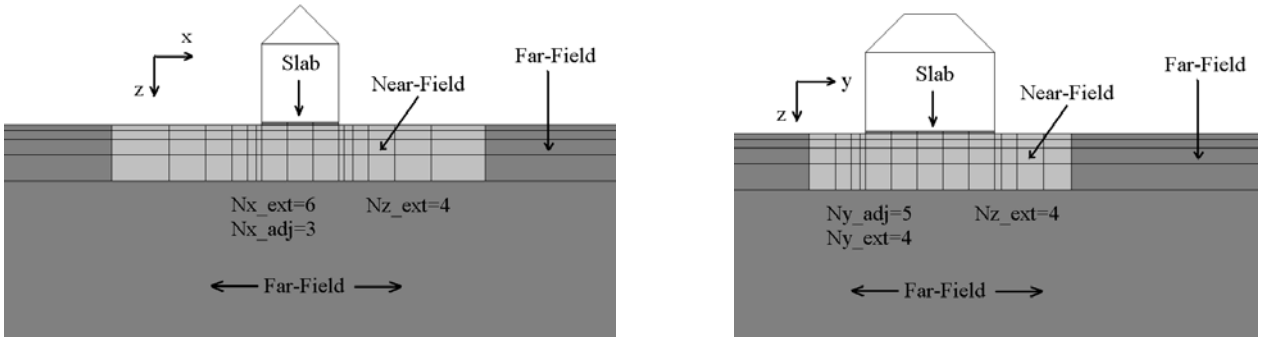


Figure 2: Slab, Near and Far Field Dimensions

The sizes of the soil nodes directly beneath the slab in the x and y-directions are automatically determined by the length and width of the slab and by the user specified number of nodes along the length and along the width of the slab. For near-field soil nodes, the user not only specifies how many nodes are to be considered in the x, y, and z-directions beyond the slab, but the length, width and depth of the nodes starting at the slab-to-soil boundary and moving outwards and downwards.

3. Ground Temperature Options

Users are asked to specify one of three near and far-field ground temperature options using Parameter 25.

The first option is to employ an energy balance on the soil surface to calculate the surface temperature. In this option, the user provides the ambient temperature, sky temperature, and the amount of incident solar radiation on the soil surface to the model as Inputs. An energy balance is applied to the surface to determine the surface temperature and conduction to the soil nodes provides the heat transfer to the ground from the surface. The Kasuda ground temperature profile [1] (which calculates a depth-dependent soil temperature based on the average annual surface temperature, the amplitude of the annual surface temperature, and the day of the year at which the minimum annual surface temperature occurs) sets the initial ground temperature profile for the near-field and for the far-field.

The second option is to use the Kasuda correlation to set the surface temperature as a function of the time of the year. In the near field, the Kasuda correlation is used to set the initial temperature profile in the soil and to obtain a time dependent surface temperature. The temperature of near field soil nodes depends upon conduction effects from neighboring nodes and from the Kasuda calculated surface temperature. In the far field, the Kasuda correlation is used to set the temperature of all nodes. The temperature of these nodes will change, but only as a function of depth and time of year.

The third option is for the user to supply the surface temperature as an Input to the model. In the near field, and in the far-field, the Kasuda correlation is used to set the initial temperature profile in the soil. The near and far-field nodes then change temperature due to conduction effect from other nodes (vertical direction only in the far-field) influenced by the surface temperature provided by the user.

4. Energy Balances

The basic energy balance on a node contained in the slab or the soil is shown in Equation 1:

$$mC_p \frac{dT}{dt} = \dot{Q}_{in} - \dot{Q}_{out} \quad \text{(Eq. 1)}$$

The equations describing Q_{in} and Q_{out} change depending upon the type of node and its location. Three types of nodes exist: internal nodes, boundary nodes (between the slab and the near field or between the near field and the far field for example), and surface nodes.

4.1 Internal Slab/Soil Nodes

If a node is contained entirely within the near-field and does not border either the far-field or the slab itself, its energy balance is made up of purely conductive terms. The equations for Q_{in} and Q_{out} are:

$$\dot{Q}_{in} = \frac{k_{soil}\, dy\, dz}{x_i - x_{i-1}}\left(\overline{T}_{x-1,y,z} - T_{x,y,z}\right) + \frac{k_{soil}\, dx\, dz}{y_i - y_{i-1}}\left(\overline{T}_{x,y-1,z} - T_{x,y,z}\right) + \frac{k_{soil}\, dx\, dy}{z_i - z_{i-1}}\left(\overline{T}_{x,y,z-1} - T_{x,y,z}\right) \quad \text{(Eq. 2)}$$

$$\dot{Q}_{out} = \frac{k_{soil}\, dy\, dz}{x_{i+1} - x_i}\left(T_{x,y,z} - \overline{T}_{x+1,y,z}\right) + \frac{k_{soil}\, dx\, dz}{y_{i+1} - y_i}\left(T_{x,y,z} - \overline{T}_{x,y+1,z}\right) + \frac{k_{soil}\, dx\, dy}{z_{i+1} - z_i}\left(\overline{T}_{x,y,z} - T_{x,y,z+1}\right) \quad \text{(Eq. 3)}$$

It should be noted that some internal soil nodes may have an additional resistance term added because Type704 allows insulation on the perimeter of the slab to extend down below the slab as far as desired.

4.2 Boundary Nodes

Two types of boundary nodes exist: nodes on the boundary between the near and far-fields, and nodes on the boundary between the slab and the near-field.

Nodes in the near-field on the boundary with the far-field are governed purely by conduction. The far-field temperature to which the node conducts is not affected by energy passing from the node to the far-field, only by time of year and depth as given by the Kasuda correlation (for mode 2) or by conduction in the vertical direction (modes 1 and 3). Nodes at the bottom corners of the near-field can conduct to two different far-field temperatures while nodes only along the depth of the near-field conduct to a single far-field temperature (along the horizontal plane). Conduction between the near and far-fields may be neglected, if desired, by setting parameters 18 and 19 to the appropriate values.

Nodes on the boundary between the slab and near-field are also governed by conduction but may have an additional pure resistance built into their energy balance. The resistive term is used to account for insulation beneath the slab.

The conduction length between the node and the far-field is one-half of the corresponding nodal distance. For example, the conduction length between a node along the bottom of the near-field and the far-field is dz/2.

4.3 Surface Nodes

Surface nodes are handled in a slightly different manner from other nodes because they interact with the surroundings, or with a known surface condition. For surface mode 1, the near-field surface temperature is calculated from an energy balance on the surface (assuming the surface has negligible mass). The energy balance on the near-field surface reveals that the energy conducted to the surface by the node must equal the energy convected (to the ambient) and radiated (to the surroundings) from the surface minus any incident radiation. The energy balance on the surface is shown schematically in Figure 3. The resultant surface temperature is used to drive the conduction heat transfer calculations into the soil nodes below.

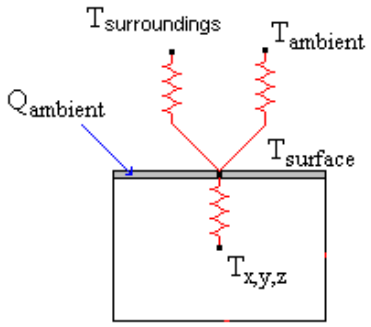


Figure 3: Energy Balance on the Near-Field Surface

The energy transferred from the near-field surface to the environment contains terms for both radiative heat transfer between the surface and the sky temperature and a convection term between the surface and the ambient. The radiation term may be linearized with ambient temperature using Equation 12.

$$h_{radiation} = \left(\varepsilon\sigma\left(T_{ambient} + T_{sky}\right)\left(T_{ambient}^2 + T_{sky}^2\right)\right) \quad \text{(Eq. 12)}$$

The energy balance on the surface is then:

$$0 = \left(h_{radiation} + h_{convection}\right)\left(T_{surface} - T_{ambient}\right) - Q_{ambient} - \frac{k\,dx\,dy\left(T_{x,y,z} - T_{surface}\right)}{dz/2} \quad \text{(Eq. 13)}$$

where $Q_{ambient}$ is the product of the incident radiation and the surface absorptance.

This energy balance is solved for the near-field surface temperature which is then used for the conduction equations for the soil/slab node.

Surface mode 2 uses the Kasuda correlation to set the surface temperature while surface mode 3 relies on an input surface temperature to the model.

For the nodes in contact with the slab, the resistance network for the calculation of the surface temperatures is shown in Figure 4. The resultant surface temperatures are then used to drive the heat transfer to the soil and the heat transfer from the surface to the zone.

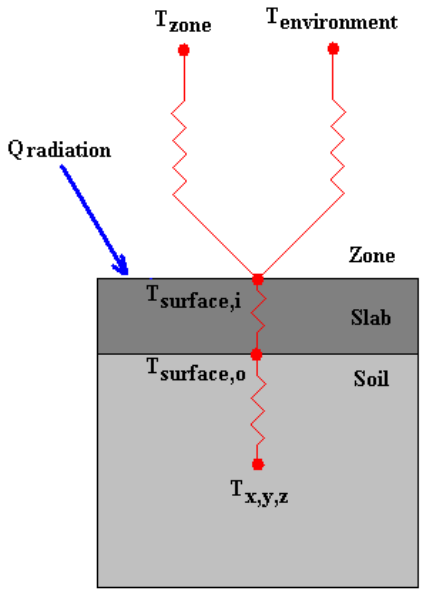


Figure 4: Energy Balance on the Slab Surface

The energy balance solution for the inner surface temperature is then:

$$T_{surface,i} = \frac{Q_{radiation} + h_{radiation}\ dx\ dy\ T_{environment} + h_{convection}\ dx\ dy\ T_{zone} + \frac{T_{x,y,z}}{R_{cond}}}{h_{radiation}\ dx\ dy + h_{convection}\ dx\ dy + \frac{1}{R_{cond}}} \qquad \text{(Eq. 14)}$$

where

$$R_{cond} = \frac{dz}{2\ k_{soil}\ dx\ dy} + \frac{1}{U_{slab}\ dx\ dy} \qquad \text{(Eq. 15)}$$

The resistance network is shown in Figure 5 for all surface soil nodes. For purposes of clarity, conduction in the y direction has been neglected from the figure. The conduction length for heat transfer to/from the surface is half the nodal height (dz/2).

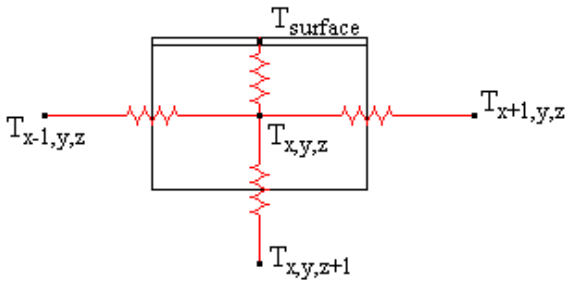


Figure 5: Energy Balance on a Surface Node

5. Model Execution Speed

Because the near-field volume of soil modeled by this component is broken up into a large number of nodes, model execution speed is often significantly reduced. Execution speed can be increased by reducing the number of nodes in the near field but this in turn sacrifices accuracy. As an alternative, the convergence tolerance on the soil temperatures contained within the near-field is provided as a PARAMETER to this model. Users may plot the energy imbalance output of the model and loosen the tolerances to increase speed while making sure that the energy imbalance does not increase inordinately.

References

[1] Kasuda, T., and Archenbach, P.R. "Earth Temperature and Thermal Diffusivity at Selected Stations in the United States", ASHRAE Transactions, Vol. 71, Part 1, 1965